

Propositional logic rules

Propositional Logic Rules Summarization

Negation of a proposition

p	$\neg p$
T	F
F	T

Conjunction of two propositions

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction of two propositions

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Exclusive OR of two propositions

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Conditional statement

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Biconditional statement

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Propositional Equivalences

Tautology and Contradiction			
p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
T	F	T	F
F	T	T	F

De Morgan's Laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$
$\neg(p \vee q) \equiv \neg p \wedge \neg q$

Logical Equivalences	
Equivalence	Name
$p \wedge T \equiv p$ $p \vee F \equiv p$	Identity laws
$p \vee T \equiv T$ $p \wedge F \equiv F$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv T$ $p \wedge \neg p \equiv F$	Negation laws

Logical Equivalences Involving Conditional Statements
$p \rightarrow q \equiv \neg p \vee q$
$p \rightarrow q \equiv \neg p \rightarrow \neg p$
$p \vee q \equiv \neg p \rightarrow q$
$p \wedge q \equiv \neg(p \rightarrow \neg q)$
$\neg(p \rightarrow q) \equiv p \wedge \neg q$
$(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$
$(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$
$(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$
$(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$

Predicates and Quantifiers

Logical Equivalences Involving Biconditional Statements

$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$

$$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$$

$$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$$

$$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$$

Quantifiers

Statements	When True?	When False?
$\forall_x P(x)$	$P(x)$ is true for every x .	There is an x for which $P(x)$ is false.
$\exists_x P(x)$	There is an x for which $P(x)$ is true.	$P(x)$ is false for every x .

De Morgan's Laws for Quantifiers

Negation	Equivalent Statement	When Is Negation True?	When False?
$\neg \exists_x P(x)$	$\forall_x \neg P(x)$	For every x , $P(x)$ is false.	There is an x for which $P(x)$ is true.
$\neg \forall_x P(x)$	$\exists_x \neg P(x)$	There is an x for which $P(x)$ is false.	$P(x)$ is true for every x .

Nested Quantifiers

Quantification of Two Variables		
Statement	When True?	When False?
$\forall x \forall y P(x, y)$ $\forall y \forall x P(x, y)$	$P(x, y)$ is true for every pair x, y .	There is a pair x, y for which $P(x, y)$ is false.
$\forall x \exists y P(x, y)$	For every x there is a y for which $P(x, y)$ is true.	There is an x such that $P(x, y)$ is false for every y .
$\exists x \forall y P(x, y)$	There is an x for which $P(x, y)$ is true for every y .	For every x there is a y for which $P(x, y)$ is false.
$\exists x \exists y P(x, y)$ $\exists y \exists x P(x, y)$	There is a pair x, y for which $P(x, y)$ is true.	$P(x, y)$ is false for every pair x, y .