Formula For the Sum Of the First N Squares - Proof

Formula for the sum of the first N squares:

$$1^2 + 2^2 + 3^2 + ... + N^2 = \frac{N(N+1)(2N+1)}{6} \tag{1}$$

Proof:

For $n = 1$, the statement reduces to $1^2 = \frac{1*2*3}{6}$ and is obviously true.

Assuming the statement is true for $n = k$:

$$1^2 + 2^2 + 3^2 + ... + k^2 = \frac{k(k+1)(2k+1)}{6}, \tag{2}$$

we will prove that the statement must be true for $n = k + 1$:

$$1^2 + 2^2 + 3^2 + ... + k^2 = \frac{(k+1)(k+2)(2k+3)}{6}, \tag{3}$$

The left-hand side of 3 can be written as

$$1^2 + 2^2 + 3^2 + ... + k^2 + (k+1)^2. \tag{4}$$

In view of 2, this simplifies to:

$$\begin{aligned}(1^2 + 2^2 + 3^2 + ... + k^2) + (k+1)^2 &= \frac{k(k+1)(2k+1)}{6} + (k+1)^2 = \\ &\frac{k(k+1)(2k+1) + 6(k+1)^2}{6} = \\ &\frac{(k+1)[k(2k+1) + 6(k+1)]}{6} = \\ &\frac{(k+1)(2k^2+7k+6)}{6} = \\ &\frac{(k+1)(k+2)(2k+3)}{6}.\end{aligned}$$

Thus the left-hand side of 3 is equal to the right-hand side of 3. This proves the inductive step. Therefore, by the principle of mathematical induction, the given statement is true for every positive integer N.