

Homework Assignment for Chapter 10 (Due by 3pm on Mar. 25)

Reference Exercise Problems: Text Book, 10.5 Exercises.

Homework problems

Problem 1 Consider the joint probability distribution of X and Y from the Problem 4 in Homework 7, obtained from data on hair color and eye color.

Eye color	Hair color		
	Fair/red	Medium	Dark/black
Light	0.22	0.1	0.1
Dark	0.48	0.1	0

- Compute $Cov(X, Y)$. Are X and Y positively correlated, negative correlated, or uncorrelated?
- Compute the correlation coefficient between X and Y

Problem 2 Consider the two discrete random variables X and Y with joint distribution derived in textbook Exercise 9.2:

b	a			
	0	1	2	$P(Y = b)$
-1	$\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{2}$
1	0	$\frac{1}{2}$	0	$\frac{1}{2}$
$P(X = a)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	1

- Determine $E[XY]$
- Note that X and Y are dependent. Show that X and Y are uncorrelated.
- Determine $Var(X + Y)$
- Determine $Var(X - Y)$

Problem 3 Let X and Y be random variables such that

$$E[X] = 3, E[Y] = 4, \text{ and } Var(X) = 5.$$

- Show that $E[X^2] = 14$.
- Determine the expectation of $-2X^2 + Y$.

Problem 4 Recall the relation between degrees Celsius and degrees Fahrenheit

$$\text{degrees Fahrenheit} = \frac{9}{5} \times \text{degrees Celsius} + 32.$$

Let X and Y be the average daily temperatures in degrees Celsius in Amsterdam and Antwerp. Suppose that $Cov(X, Y) = 2$ and $\rho(X, Y) = 0.6$. Let T and S be the same temperatures in degrees Fahrenheit. Compute $Cov(T, S)$ and $\rho(T, S)$.

Problem 5 In Figure 1 three plots are displayed. For each plot we carried out a simulation in which we generated 500 realizations of a pair of random variables (X, Y) . We have chosen three different joint distributions of X and Y .

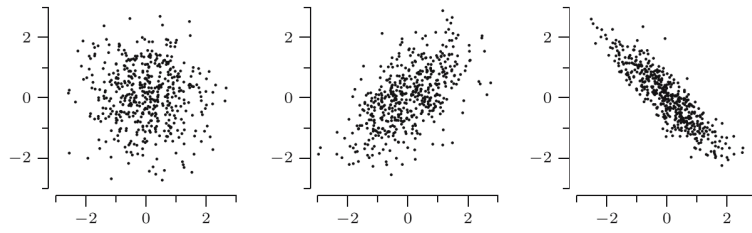


Figure 1: Some scatterplots.

- a. Indicate for each plot whether it corresponds to random variables X and Y that are positively correlated, negatively correlated, or uncorrelated.
- b. Which plot corresponds to random variables X and Y for which $|\rho(X, Y)|$ is maximal?