Homework Assignment for Chapter 7 (Due by 3pm on Feb. 25)

Reference Exercise Problems: Text Book, 7.6 Exercises.

Homework problems

Problem 1 Let T be the outcome of a roll with a fair seven sided die.

a) Describe the probability distribution of T , that is, list the outcomes and the corresponding probabilities.

b) Determine $E[T]$ and $Var(T)$.

Problem 2 The probability distribution of a discrete random variable X is given by

$P(X = -1) = \frac{2}{12}$, $P(X = 0) = \frac{7}{12}$, $P(X = 1) = \frac{3}{12}$.

a) Compute $E[X]$.

b) Give the probability distribution of $Y = X^2$ and compute $E[Y]$ using the distribution of Y.

c) Determine $E[X^2]$ using the change-of-variable formula. Check your answer against the answer in **b**.

d) Determine $Var(X)$.

Problem 3 Given is a random variable X with probability density function f given by $f(x) = 0$ for $x < 0$, and for $x > 1$, and $f(x) = 4x^2 - 4x^3$ for $0 \leq x \leq 1$. Determine the expectation and variance of the random variable $2X + 3$.

Problem 4 Let U be a random variable with a $U(\alpha, \beta)$ distribution.

a) Determine the expectation of U.

b) Determine the variance of U.

After expressing expectation and variance of U, apply it on well known $U(0, 1)$ distribution.

Problem 5 Let X be a random variable and r and s any real numbers. Use the change-of-units rule $E[rX + s + t^2] = rE[X] + s + t^2$ for the expectation to obtain **a** and **b**.

a) Show that $Var(rX) = r^2 Var(X)$.

b) Show that $Var(X + s + t^2) = Var(X)$.

c) Combine parts **a** and **b** to show that

$$Var(rX + s + t^2) = r^2 Var(X).$$