

Homework Assignment for Chapter 4 (Due by 3pm on Feb. 4)

Reference Exercise Problems: Text Book, 4.6 Exercises.

Homework problems

Problem 1 Let X be a discrete random variable with probability mass function p given by:

a	-1	0	1	2
$p(a)$	1/12	1/2	1/4	1/6

and $p(a) = 0$ for all other a .

- Let the random variable Y be defined by $Y = X^2$, i.e., if $X = 2$, then $Y = 4$. Calculate the probability mass function of Y .
- Calculate the value of the distribution functions of X and Y in $a = 1$, $a = 3/4$, and $a = \pi - 3$.

Problem 2 Suppose that the distribution function of a discrete random variable X is given by

$$F(a) = \begin{cases} 0 & \text{for } a < 0 \\ \frac{1}{2} & \text{for } 0 \leq a < \frac{1}{3} \\ \frac{2}{3} & \text{for } \frac{1}{3} \leq a < \frac{2}{3} \\ 1 & \text{for } a \geq \frac{2}{3} \end{cases} \quad (1)$$

Determine the probability mass function of X .

Problem 3 You toss n coins, each showing heads with probability p , independently of the other tosses. Each coin that shows tails is tossed again. Let X be the total number of tails.

- What type of distribution does X have? Specify its parameter(s).
- What is the probability mass function of the total number of tails X ?

Problem 4 A shop receives a batch of 750 cheap lamps. The odds that a lamp is defective are 0.5%. Let X be the number of defective lamps in the batch.

- What kind of distribution does X have? What is/are the value(s) of parameter(s) of this distribution?
- What is the probability that the batch contains no defective lamps? One defective lamp? More than one defective one?

Problem 5 You decide to play monthly in three different lotteries, and you stop playing as soon as you win a prize in one (or in all three) lotteries of at least one million euros. Suppose that every time you participate in these lotteries, the probability to win one million (or more) euros is p_1 for one of the lotteries and p_2 for the other and p_3 for last one. Let M be the number of times you participate in these lotteries until winning at least one prize. What kind of distribution does M have, and what is its parameter?