Homework Assignment for Chapter 3 (Due by 3pm on Jan. 28)

Reference Exercise Problems: Text Book, 3.6 Exercises.

Homework problems

Problem 1 A Dutch cow is tested for BSE, using Test A as described in Text Book, Section 3.3 , with $P(T \mid B) = 0.70$ and $P(T \mid B^c) = 0.10$. Assume that the BSE risk for the Netherlands is the same as in 2003, when it was estimated to be $P(B) = 2.7 * 10^{-5}$. Compute $P(B \mid T)$ and $P(B \mid T^c)$.

Problem 2 Calculate:

1. $P(A \cup B)$ if it is given that $P(A) = 1/2$ and $P(B \mid A^c) = 3/5$.
2. $P(B)$ if it is given that $P(A \cup B) = 5/6$ and $P(A^c \mid B^c) = 2/6$.

Problem 3 A certain grapefruit variety is grown in two regions in southern Spain. Both areas get infested from time to time with parasites that damage the crop. Let A be the event that region R_1 is infested with parasites and B that region R_2 is infested. Suppose $P(A) = 1/3$, $P(B) = 2/3$ and $P(A \cup B) = 14/15$. If the food inspection detects the parasite in a ship carrying grapefruits from R_1, what is the probability region R_2 is infested as well?

Problem 4 A breath analyzer, used by the police to test whether drivers exceed the legal limit set for the blood alcohol percentage while driving, is known to satisfy

$P(A \mid B) = P(A^c \mid B^c) = p,$

where A is the event "breath analyzer indicates that legal limit is exceeded" and B "driver's blood alcohol percentage exceeds legal limit." On Saturday night about 10% of the drivers are known to exceed the limit.

1. Describe in words the meaning of $P(B^c|A)$.
2. Determine $P(B^c|A)$ if $p = 0.85$.
3. How big should p be so that $P(B \mid A) = 0.95$?

Problem 5 You are diagnosed with an uncommon disease. You know that there only is a 0.5% chance of getting it. Use the letter D for the event "you have the disease" and T for "the test says so." It is known that the test is imperfect:

$P(T \mid D) = 0.95$ and $P(T^c \mid D^c) = 0.91$.

1. Given that you test positive, what is the probability that you really have the disease?
2. You obtain a second opinion: an independent repetition of the test. You test positive again. Given this, what is the probability that you really have the disease?