Extra Homework Assignment (Due by 1pm on Apr. 27)

Homework problems

Problem 1 Suppose the random variables X_1, X_2,..., X_n have the same expectation μ. For which constants a and b is $T = a(X_1 + X_2 + ... + X_n) + b$ an unbiased estimator for μ?

Problem 2 Given are two estimators S and T for a parameter Θ. Furthermore it is known that $Var(S) = 40$ and $Var(T) = 4$.

a. Suppose that we know that $E[S] = \Theta + 10$ and $E[T] = \Theta + 3$. Which estimator would you prefer, and why?

b. Suppose that we know that $E[S] = \Theta + 10$ and $E[T] = \Theta + a$ for some positive number a. For each a, which estimator would you prefer, and why?

Problem 3 Given is a random sample X_1, X_2,..., X_n from a $Ber(p)$ distribution. One considers the estimators $T_1 = \frac{1}{n}(X_1 + ... + X_n)$ and $T_2 = min\{X_1, ..., X_n\}$.

a. Are T_1 and T_2 unbiased estimators for p?

b. Show that $MSE(T_1) = \frac{1}{n}p(1-p)$, $MSE(T_2) = p^n - 2p^{n+1} + p^2$.

c. Which estimator is more efficient when $n = 2$?

Problem 4 In Homework 11 problem 1 we modeled the hits of London by flying bombs by a Poisson distribution with parameter μ.

a. Use the data from Homework 11 problem 1 to find the maximum likelihood estimate of μ.

b. Suppose the summarized data from Homework 11 problem 1 got corrupted in the following way:

Number of hits	0 or 1	2	3	4	5	6	7
Number of squares	431	93	35	7	8	1	1

Using this new data, what is the maximum likelihood estimate of μ?

Problem 5 Suppose that x_1, x_2, ..., x_n is a dataset, which is a realization of a random sample from a normal distribution.

Let the probability density of this normal distribution be given by $f_\mu(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}(x-\mu)^2}$ for $-\infty < x < \infty$. Determine the maximum likelihood estimate for μ.

Problem 6 Let x_1, x_2, ..., x_n be a dataset that is a realization of a random sample from a distribution with probability density $f_\delta(x)$ given by

$$f_\delta(x) = \begin{cases} e^{-(x-\delta)} & \text{for} x \geq \delta \\ 0 & \text{for} x < \delta \end{cases}$$

a. Draw the likelihood $L(\delta)$.

b. Determine the maximum likelihood estimate for δ.