Conditional probability and independence

- Define two events A and B.

Conditional probability

$$P(A \cap B) = P(A \mid B)P(B) = P(B \mid A)P(A) \tag{1}$$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} \text{ or } P(B \mid A) = \frac{P(B \cap A)}{P(A)}$$

The law of total probability

- Suppose $B_1, B_2, \ldots, B_N$ are disjoint events and that $B_1 \cup B_2 \cup \ldots \cup B_N = \Omega$

$$P(A) = P(A \mid B_1)P(B_1) + P(A \mid B_2)P(B_2) + \ldots + P(A \mid B_N)P(B_N) \tag{2}$$

Bayes' rule

$$P(B_i \mid A) = \frac{P(A \mid B_i)P(B_i)}{P(A \mid B_1)P(B_1) + P(A \mid B_2)P(B_2) + \ldots + P(A \mid B_N)P(B_N)} \tag{3}$$

Independence

$$P(A \mid B) = P(A) \text{ and } P(B \mid A) = P(B) \quad \text{and } P(B \cap A) = P(B)P(A)$$