Homework Assignment for Chapter 9 (Due by 3pm on Mar. 11)

Reference Exercise Problems: Text Book, 9.7 Exercises.

Homework problems

Problem 1 The joint probabilities $P(X = a, Y = b)$ of discrete random variables X and Y are given in the following table (which is based on the magical square in Albrecht Durer's engraving Melencolia I in Figure 1). Determine the marginal probability distributions of X and Y , i.e., determine the probabilities $P(X = a)$ and $P(Y = b)$ for $a, b = 1, 2, 3, 4$.

Figure 1: Albrecht Durer's Melencolia I. Albrecht Durer (German, 1471-1528) Melencolia I, 1514. Engraving. Bequest of William P. Chapman, Jr., Class of 1895. Courtesy of the Herbert F. Johnson Museum of Art, Cornell University.

	a			
b	1	2	3	4
1	13/136	2/136	3/136	16/136
2	9/136	6/136	7/136	12/136
3	5/136	10/136	11/136	8/136
4	1/136	15/136	14/136	4/136

Problem 2 Let X and Y be two random variables, with joint distribution the Melencolia distribution, given by the table in Problem 1. What is

a. $P(X = Y)$?

b. $P(X + Y = 5)$?

c. $P(1 < X \leq 3, 1 < Y \leq 3)$?

d. $P((X, Y) \in \{1, 4\} \times \{1, 4\})$?

Problem 3 Let X and Y be two independent $Ber(\frac{1}{2})$ random variables. Define random variables U and V by:

$$U = (X+Y)^2 + 1 \text{ and } V = |X - Y|.$$

a. Determine the joint and marginal probability distributions of U and V.

b. Find out whether U and V are dependent or independent.

Problem 4 To investigate the relation between hair color and eye color, the hair color and eye color of 5046 persons was recorded. The data are given in the following table:

	Hair color		
Eye color	Fair/red	Medium	Dark/black
Light	1123	495	845
Dark	2500	78	5

Source: B. Everitt and G. Dunn. Applied multivariate data analysis. Second edition Hodder Arnold, 2001; Table 4.12. Reproduced by permission of Hodder & Stoughton.

Eye color is encoded by the values 1 (Light) and 2 (Dark), and hair color by 1 (Fair/red), 2 (Medium), and 3 (Dark/black). By dividing the numbers in the table by 5046, the table is turned into a joint probability distribution for random variables X (hair color) taking values 1 to 3 and Y (eye color) taking values 1 and 2.

a. Determine the joint and marginal probability distributions of X and Y.

b. Find out whether X and Y are dependent or independent.

Problem 5 Let X and Y be independent random variables with probability distributions given by $P(X = 0) = P(X = 1) = \frac{1}{2}$ and $P(Y = 0) = P(Y = 2) = \frac{1}{2}$.

a. Compute the distribution of $Z = (X - Y)^2$.

b. Let $\overline{Y}$ and $\overline{Z}$ be independent random variables, where $\overline{Y}$ has the same distribution as Y , and $\overline{Z}$ the same distribution as Z. Compute the distribution of $\overline{X} = \overline{Z} - \overline{Y}$.