Reference Exercise Problems: Text Book, 2.7 Exercises.

## Homework problems

**Problem 1** Let A and B be two events in a sample space for which P(A) = 1/5, P(B) = 4/6, and  $P(A \cap B) = 2/3$ . What is  $P(A \cup B)$ ?

**Problem 2** Let *E* and *F* be two events for which one knows that the probability that at least one of them occurs is 1/8. What is the probability that neither *E* nor *F* occurs? Hint: use one of DeMorgan's laws:  $E^c \cap F^c = (E \cup F)^c$ .

**Problem 3** We consider events A, B, and C, which can occur in some experiment. Is it true that the probability that only A occurs (and not B or C) is equal to  $P(A \cup B \cup C) - P(B) - P(C) + P(B \cap C)$ ?

**Problem 4** We toss a coin three times. For this experiment we choose the sample space  $\Omega = \{HHH, THH, HTH, HHT, TTH, THT, HTT, TTT\}$  where T stands for tails and H for heads.

- 1. Write down the set of outcomes corresponding to each of the following events:
  - (a) A : "we had tails exactly two times."
  - (b) B : "we throw tails at least one time."
  - (c) C: "head did not appear before a tails appeared."
  - (d) D: "the first throw results in head."
- 2. Write down the set of outcomes corresponding to each of the following events:  $A^c$ ,  $D^c$ ,  $A \cup (C \cap D)$ , and  $A \cap B^c$ .

**Problem 5** In some experiment first an arbitrary choice is made out of five possibilities, and then an arbitrary choice is made out of the remaining four possibilities. With a rule that if a was chosen first, only b and c may be chosen next. Likewise, if d is chosen first only a can follow it. One way to describe this is with a product of two sample spaces  $\{a, b, c, d, e\}$ :

 $\Omega = \{a, b, c, d, e\} \times \{a, b, c, d, e\}.$ 

- 1. Make a 5  $\times$  5 table in which you write the probabilities of the outcomes.
- 2. Describe the event "c is one of the chosen possibilities" and determine its probability.